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A method for comparing the performance of open channel velocity-area flow meters and critical depth flow meters

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Abstract

New technologies enable velocity measurements to be acquired continuously from a moving body of water flowing on an open channel. They provide an alternative to the well-established flow measurement methods using weirs and flumes: commonly known as the critical-depth methods. These velocity measurements must be integrated across the measurement cross-section to enable the flow rate to be calculated. Open channel flow is turbulent and therefore the measurement process needed to determine the average velocity must be complex. At present, there is little or no independent data to define the measurement performance of velocity-area techniques. The critical-depth method, however, has been thoroughly researched and its performance is well defined in the various published Hydrometry Standards. Using the critical-depth method as a benchmark, measurement uncertainty analysis is used to define performance criteria required of velocity-area methods.

1. Introduction

The direct method of measurement of flow in open channels requires i) the measurement of the mean of the velocity across the channel section and ii) the measurement of the cross-section area through which that velocity passes. The product of these two quantities is equal to the rate of flow.

The cross-section area is determined from a knowledge of the channel geometry and from a measurement of the depth of water. If the channel is man-made, this can usually be done with a measurement uncertainty of 2%.

Nomenclature

u^*	dimensionless standard uncertainty of a variable ($b,h,\bar{V},etc.$) (usually expressed as a percentage)
u^*uc	combined dimensionless standard uncertainty of a variable ($b,h,\bar{V},etc.$) (usually expressed as a percentage)
b	width dimension of a rectangular channel
b'	width dimension of a contracted section of a rectangular channel (flume throat)
h	depth (head) of water in a channel
h'	depth of water in a contracted section of a channel
h_C	depth of water in a contracted section of a channel at the critical condition
H	total head of water in the channel
C_D	discharge coefficient of flow through a weir or flume
Q_C	flow rate in the channel determined by the critical depth method
Q_{VA}	flow rate in a channel determined by the velocity area method#

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The technical challenge is to find the mean velocity. Friction at the channel walls causes strong velocity gradients, illustrated in Fig. 1, which are unstable and migrate as vortices through the body of the flow. This causes turbulence and unsteady conditions. (Note, turbulence exists in a moving body of water even when the water surface appears tranquil.) The measurement process therefore needs to scan the cross-section while integrating and averaging the velocity components.

Technologies used for the direct method include: time-of-flight ultrasonics, pulsed Doppler sonar and electromagnetic methods. More recently, Doppler radar has been used.

Through the guidelines of [1] it is possible to define criteria for the comparison of the measurement performance of these techniques. The superior technique would be the one that minimises the uncertainty $u^*(\bar{V})$ of the mean velocity $\bar{V}$ of the turbulent profiles illustrated in Fig. 1.

2. Weirs and flumes—a benchmark technology

Direct methods can be compared as a class with the well-established critical-depth method (the basis of the weir and flume technique).

Since the 19th century it has been known that when flow passes over a weir, a unique relationship exists between the upstream water level and the flow rate, and that relationship is largely independent of the velocity profiles approaching the weir. Analysis shows that by accelerating the flow at a weir or through a flume, the velocity distortions are greatly reduced and that, for practical purposes, the velocities adopt a geometrically consistent pattern for each class of weir or flume. This is shown in Fig. 2 for a rectangular flume.

A unique relationship therefore exists between the upstream water level, the cross-section of the accelerated flow and the mean velocity in the accelerated section. This relationship is defined by the critical depth theory.

Weirs and flumes have a long history of laboratory investigation. The uncertainties of the measurement process have been carefully researched so that the ISO Standards now include procedures for the evaluation of uncertainty. These procedures can be used to establish a 'benchmark' for the assessment of direct velocity-area methods.

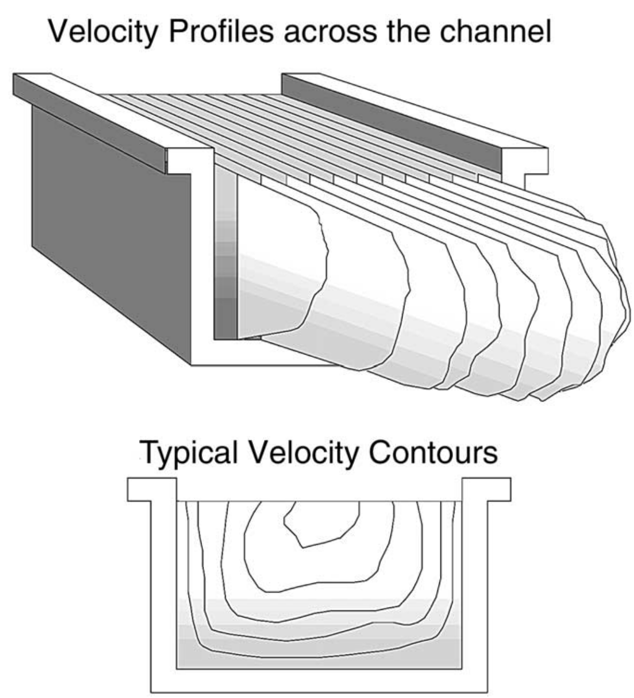

Fig. 1. Typical velocity profiles and contours

3. Measurement uncertainty

There are various rules that can be applied to any measurement process to state the quality of the results in terms of uncertainty. A flow measurement can never be exact. For example if water is controlled to flow at a constant rate, then a flow meter will exhibit a spread of measurements about a mean value. The standard deviation of this spread of measurements is, by definition, termed standard uncertainty.

The standard deviation of a set of measurements can be directly used to estimate the uncertainty of velocity or head measurements (the Type-A methods of [1]).

This would be inappropriate for measuring the channel geometry. An alternative to working with standard deviation is to define a probability distribution for a measurement process.

The GUM [1] and ISO 5168 [2] provide guidance on the application of the principles of measurement uncertainty. These documents develop the concept of standard uncertainty to include:

1. standard deviation of the mean value of a set of measurements

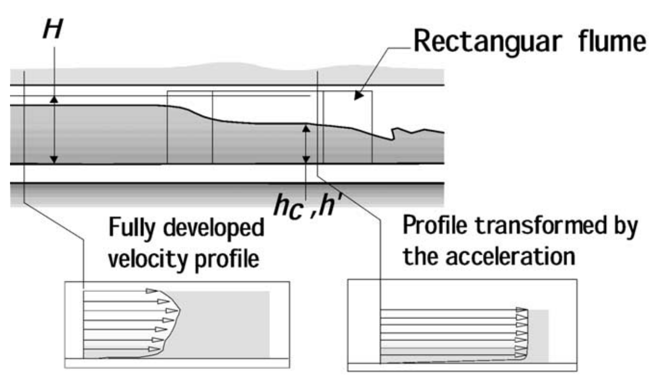

Fig. 2. Acceleration through a flume

2. probability distributions for simple measurement processes to enable the equivalent standard uncertainty values to be estimated (the Type-B methods of [1]), and
3. how to combine the uncertainties of the variables in the formula to derive flow rate for each class of weir or flume.
4. how to expand uncertainty estimations from standard values to values at the 95% confidence limit.

An analysis of flow measurement uncertainty starts with the formula used for computation.

4. Formula for the computation of flow in rectangular channels

4.1. The direct-method velocity-area equation (rectangular cross-sections)

$$Q_{VA} = b \times h \times \bar{V} \tag{1}$$

This equation defines the flow rate Q through a rectangular channel of width b and water depth h. The most problematic of these is the measurement of mean velocity $\bar{V}$ which is known to vary strongly across the channel cross-section (see Fig. 1).

4.2. The critical-depth equation (rectangular cross-sections)

This variant of the basic equation relates the mean velocity $\bar{V}$ to the change in the water surface level that occurs when the flow is accelerated in the channel. Here, the acceleration is induced by a contraction of the width of the channel (such as a long-throat flume). If the streamlines within the contraction have very little curvature, then it can be shown that $\bar{V} = \sqrt{2g(H-h')}$ where H is the total head of the flow in the channel and h' is the head of water in the contraction, refer to Fig. 2.

Thus;

$$Q_C = b' \times h' \times \sqrt{2g(H-h')}$$

where b' is the width of the rectangular channel in the contracted section.

Critical depth theory shows that for a rectangular cross-section, the head of water h' in the contraction can be reduced only to a limiting value h_C known as the critical depth which is related to the total head H by

$$h_C = \frac{2}{3}H$$

Therefore

$$Q_C = b' \times \frac{2}{3}H \times \sqrt{2g\left(\frac{1}{3}H\right)}$$

This equation is exactly equivalent to (1) with the depth and velocity terms replaced by $2H/3$ and $\sqrt{2gH/3}$ respectively.

To account for factors not included in this simplified theory, for example curvature of streamlines over a weir or the development of boundary layers in flumes, a discharge coefficient C_D is introduced. Thus, for rectangular cross-sections,

$$Q_C = C_D \times b' \times \frac{2}{3}H \times \sqrt{\frac{2g}{3}(H)}$$

This equation is usually presented in the form:

$$Q_C = \left(\frac{2}{3}\right)^{1.5} \sqrt{g} \times C_D \times b' \times H^{1.5} \tag{2}$$

For rectangular weirs, the value of C_D is determined from laboratory tests, the results of which are presented in the various ISO standards. For rectangular long-throat flumes, the value of C_D can be reliably predicted by the application of boundary-layer theory [3,4].

Note. This analysis uses the assumption that H is constant in the channel whereas in reality, it varies slightly across the approach section. The magnitude of the variation however is small compared with the mean value of H.

5. Uncertainty estimation of flow measurement

References [1] and [2] describe the relationship between the variables of Eqs. (1) and (2) and their respective measurement uncertainties. The relationships are:

$$u^*_C(Q_{VA}) = \sqrt{u^*(b)^2 + u^*(h)^2 + u^*(\bar{V})^2} \tag{3}$$

$$u^*_C(Q_C) = \sqrt{u^*(C_D)^2 + u^*(b')^2 + (1.5u^*(H))^2} \tag{4}$$

These equations show how the combined uncertainties $u^*_C(Q_{VA})$ and $u^*_C(Q_C)$ are related to the uncertainty of the variables of their respective equations $u^*(b)$,$u^*(C_D)$ etc. An error in any one of the components will induce a corresponding percentage error in Q.

Note that Eq. (4) for critical depth methods includes H to the power 1.5 which makes Q more sensitive to error in H than when used in the velocity-area Eq. (3). The sensitivity is the amount of change of Q that occurs for any given change of H, i.e. the rate of change of Q with respect to H, which is $\frac{\partial Q}{\partial H}$. From (2) this value for a rectangular flume is 1.5. Therefore, the critical depth method of flow measurement is one and a half times more sensitive to errors of head measurement than the direct methods using the velocity-area equation.

Sensitivities of the flow value with respect to errors of

b measurement are the same for velocity-area and critical depth methods. This also applies to errors in $\bar{V}$ and C_D.

6. The measurement performance of velocity-area methods compared with critical depth methods

The measurement performances of weirs and flumes are well established and documented in Standards. $u^*_C(Q_C)$ can therefore be used as the benchmark for the comparison. The condition for velocity-area methods to have better measurement performance is:

$$u^*_C(Q_{VA}) < uc^*(Q_C)$$

Using Eqs. (3) and (4)

$$u^*(b)^2 + u^*(h)^2 + u^*(\bar{V})^2 < u^*(C_D)^2 + u^*(b')^2 + (1.5u^*(H))^2 \quad (5)$$

Assuming that the same measurement methods for width and head are used throughout then it is reasonable to assume that the evaluations of uncertainty will be similar. It is therefore assumed to a first approximation that:

$$u^*(b) \doteq u^*(b')$$

$$u^*(h) \doteq u^*(H)$$

So Eq. (5) can be rewritten,

$$u^*_C(\bar{V}) < \sqrt{u^*(C_D)^2 + 1.25u^*(H)^2} \quad (6)$$

The significance of Eq. (6) is illustrated in the following example with a typical flume of throat width 0.400 m with a maximum head of water in the approach channel of 0.600 m. It is assumed that the head measurement carries an uncertainty of 0.003 m.

Fig. 3 is a graph of head measurement uncertainty and discharge coefficient uncertainty against flow rate. Flow rate and $u^*(C_D)$ have been calculated using the methods of reference [3].

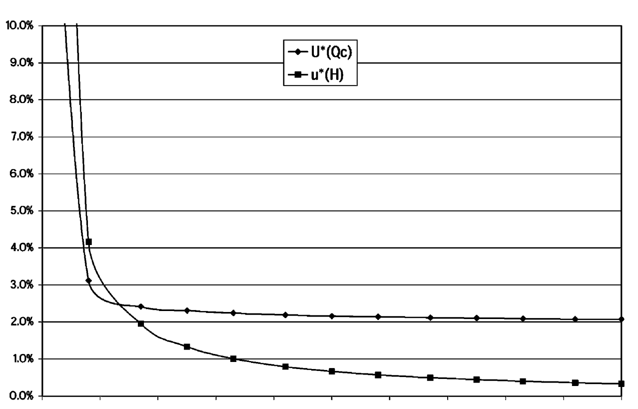

Fig. 3. Typical variation of head and discharge coefficient uncertainty with flow rate

This data is used in (6) to define the minimum criteria for $u^*_C(\bar{V})$. This is shown in Fig. 4.

Velocity-area methods able to measure $\bar{V}$ with uncertainty values below the curve would outperform weirs and flumes; those above the curve would not.

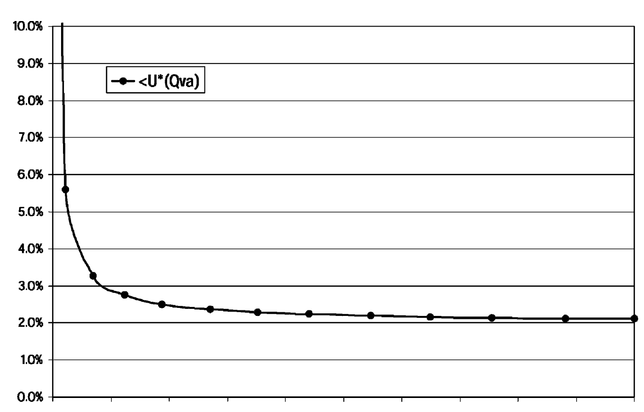

Fig. 4. Minimum criteria for velocity measurement uncertainty

7. Discussion of velocity measurement technologies

Ideally, a velocity-area method should scan the channel cross-section rapidly to obtain a 'snapshot' of the velocity profile. Assuming that the velocities are accurate to 1%, and the ability to resolve spatially (locate velocity contours) is similarly accurate then the integration process should be able to derive the mean velocity to better than 2%.

In practice, the methods are less rigorous. This is discussed briefly below.

7.1. Electromagnetic methods [6]

An electromagnetic field is used to induce a voltage gradient across the channel which is detected by electrodes on opposite walls. The induced voltage is related to the integrated effect of the velocity components crossing a path between the electrodes.

The electrode voltage is not uniquely related to the mean velocity by a simple formula. The relationship depends on the construction of the metering system itself, the location of the electrodes relative to the water surface and other factors. To resolve this, electromagnetic meters are individually calibrated.

7.2. Doppler sonar [7]

High frequency sonar reflected from particles moving with the water cause Doppler-shifted echoes. When transmitted in short bursts, the reflections can be detected at varying distances along the sonic path to define a velocity profile. There are technical problems associated with this method:

1. short pulses are needed to provide good spatial resolution, but short pulses give poor velocity resolution
2. turbulence and velocity gradients 'blur' the reflected signals
3. relationship between the reflected signal strength, the distance along the path and the particle size is unpredictable.
4. Sonar side-lobes prevent measurement along paths close to the channel walls.

Each of these factors carries a portion of uncertainty.

7.3. Transit-time Sonar [5]

Sonar transceivers are arranged to propagate ultrasonic pulses along a path across the channel angled to the direction of the flow. There is a unique relation between the following i) the propagation angle, ii) the difference between the transit times of pulses directed with and against the flow, iii) the channel width, and iv) the mean velocity of the streamlines intersecting the path. The mean channel velocity can be determined by using a large number of paths.

Unlike electromagnetic and Doppler methods, this technique provides a direct measure of mean velocity along the path. It therefore requires no calibration. The transit-time method has the potential to measure mean velocity measurements with an uncertainty order of 2%.

In practice, a small number of paths are used. Therefore assumptions are made of the velocity profiles between the paths which introduces a portion of uncertainty over and above those related to the angle, timing and distance measurements (listed above). The main difficulty lies in the application of the technique to small channels. The pulse time differential becomes very small, especially for low velocities. Path distortion can also be problematic in shallow channels.

8. Conclusions

The criterion of Eq. (6) applies to velocity measurement techniques in rectangular channels and is compared with measurements using rectangular flumes. Similar criteria apply to flume and weir types, the rectangular form being chosen as representative of all critical depth applications.

Improvements in level measurement technology are likely to reduce the value of $u(h)$ to values of the order of 0.001 m. In which case, the target performance criteria for $u_C^*(\bar{V})$ will be determined largely by the published values of $u_C^*(C_D)$: currently with measurement uncertainties of the order of 2–3%.

To compete, velocity-area methods must be capable of demonstrating velocity integration across a channel with similar levels of measurement uncertainty.

Criteria derived from this analysis present a challenge to the various velocity-area methods which, to outperform measurement using the critical depth method, must determine the value of $\bar{V}$ with a target uncertainty of between 2 and 3%.

Acknowledgements

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